A DETERMINISTIC SOLUTION FOR FORCE-USING BILATERAL MONOPOLISTS

Johnnie B. Linn III
Division of Business
Concord University
PO Box 1000, Campus Box 56
Athens, WV 24712

ABSTRACT
The traditional solution for bilateral monopoly (a monosony employer versus a monopsony union) is indeterminate. Furthermore, cost functions for labor unions are not well defined. This paper, by introducing use of force for the bilateral monopolists, solves the cost function identification problem for the union and provides for a unique solution of the bilateral monopoly equilibrium. A profit-maximizing monopsonist hires workers and users of force from a local isolated population. The guards are used against a union, which has hired users of force to extract a public good, such as improvement of working conditions, from the employer. As a result of the demand for users of force, the employer faces a decrease in the supply of traditional workers available to it and a corresponding increase of their marginal resource cost. The wage finds equilibrium where the compensation of workers has parity with that of the newly-hired users of force.

INTRODUCTION
In a previous paper, Linn (2006) explored conditions under which oligopolistic firms that produce both output and force can exist in a steady-state equilibrium in the absence of outside intervention. In the current paper the analysis is modified to bilateral monopoly. After discussing modifications to the general model for bilateral monopoly, the paper will offer two examples: a coal camp and professional sports.

THE GENERAL MODEL
For readers not having the previous paper in hand, the underlying model is reiterated in this section. Firms produce and fight over assets in absence of government. A force technology links effort to force, and an allocation rule links force to winnings. The force technology and allocation rule constitute a division of labor in the mathematics and bring about a more efficient model than does a single equation linking effort to winnings. A ratio rule is assumed, under which winnings are exhaustive and the share of winnings to the \(i^{th}\) agent employing force \(F_i\) is

\[
\phi_i = \frac{F_i}{\sum F_j}.
\]

Two agents use their force and output technologies to fight over their pooled output and generate winnings

\[
W_i = p\phi_i(Y_i + Y_j), \quad i = 1, 2
\]

The outputs of the two agents are assumed to be homogenous, and output price \(p\) is a function of joint output. The two items produced are force and output, assumed to be flow variables, produced by homogenous, divisible labor that can be employed either as "workers" or "guards." Unemployed individuals constitute "outliers" who use force but who do not act collectively on the margin.

The own-force elasticity of a user’s share of winnings is

\[
\frac{F_i}{\phi_i} \frac{\partial \phi_i}{\partial F_i} = 1 - \phi_i,
\]

and the cross-force elasticity of winnings is

\[
\frac{F_i}{\phi_i} \frac{\partial \phi_i}{\partial F_j} = -\phi_i, \quad i \neq j.
\]

The production functions for the parties’ output and force in elasticized form are

\[
L \frac{\partial Y}{\partial L} = \alpha,
\]

and

\[
\frac{G_i}{F_i} \frac{\partial F_i}{\partial G_i} = \beta_i, \quad i = 1, 2.
\]

SPECIAL ASSUMPTIONS FOR THE BILATERAL MONOPOLY MODEL
The special assumptions for the bilateral monopoly are three:

1. The monopsonist, the employer, is the sole producer of output.
2. The monopolist, the union, seeks to extract concessions that constitute a public good for the membership of the union. The public good and the wages paid to workers are the compensation package for the union.

3. The value of the public good is to be equivalent to the amount of the employer’s gross output that the union can extract by force.

In a steady-state model, there are no strikes. In a strike scenario, the employer would bring in strikebreakers and guards, and the workers would bring in flying pickets and guards of their own. In this model, these are replaced with guards on permanent employment with either party.

**EQUILIBRIUM CONDITIONS FOR THE PARTIES**

We will emphasize the conflict of labor and management in starkest terms by assuming that guards are hired only to use force against workers, not to protect the employer’s output, that is, we will assume that outliers are absent.

The profit maximization function for the employer is

\[ \pi = \phi(pY + p_0Y_0) - w(M)L - wG_1 \]  

where \( M \) is the amount of labor in the arena, \( L \) and \( G_1 \) are workers and guards, respectively, hired by the firm, \( Y_0 \) is the quantity of the union-extracted public good expressed at its marginal rate of transformation with the output good, and \( p_0 \) is the marginal valuation that the union gives to the public good.

The wages paid to different occupations of labor are not necessarily the same, but subscripts for the wages will not be carried in the equations because any given wage is identifiable from the variable it is associated with.

Since the employer will not be surrendering actual output to the union, the equilibrium will require that the employer’s share of output won be equal to the amount it produced, or

\[ \phi(pY + p_0Y_0) = Y \]  

An additional condition to be imposed at equilibrium is that

\[ p_0 = p \]  

The price seen by the union for the output good is the marginal benefit it receives from the public good at its marginal rate of transformation from the output good, but if this price is not at parity with the price of the output good the employer and union will negotiate a change in the volume of the public good. If the marginal benefit of the public good to the union is less than the output price, the union will demand a cash concession from the employer in exchange for reduction in the amount of the public good that the union will extract by force. If the marginal benefit of the public good to the union is greater than the output price, the union will offer a cash concession to the employer in exchange for an increase in the public good. In equilibrium, the marginal valuation of the output good will be at par for both parties.

The employer’s first-order condition for its workers is

\[ \frac{\alpha \phi(1 - \lambda)pY}{\lambda} - w(1 + \sigma \kappa) = 0 \]  

where \( \lambda \), or the Lerner index, is the negative inverse of the elasticity of demand for the output good, \( \sigma \) is the proportion of labor in the arena employed as workers, and \( \kappa \) is the inverse of the elasticity of supply of labor to the arena.

The employer’s first-order condition for its guards is

\[ \frac{\beta \phi(1 - \lambda)(pY + p_0Y_0)}{G_1} - w(1 + \rho_1 \kappa) = 0 \]  

where \( \rho_1 \) is the proportion of labor in the arena employed as the firm’s guards.

The union is to maximize its net winnings

\[ \pi_2 = \phi(pY + p_0Y_0) - wG_2. \]  

where \( G_2 \) is the number of guards hired by the union. The first-order condition for the union’s guards is

\[ \frac{\beta_2 \phi(1 - \lambda)(pY + p_0Y_0)}{G_2} - w(1 + \rho_2 \kappa) = 0 \]  

where \( \rho_2 \) is the proportion of labor in the arena hired as the union’s guards.

In the absence of outliers, the employer and the union absorb all of the arena output, so a convenient cancellation of terms occurs when Equations (11) and (13) are combined, as follows:

\[ \frac{w(1 + \rho_2 \kappa)G_2}{w(1 + \rho_1 \kappa)G_1} = \frac{\beta_2}{\beta_1} \]  

When \( G_2 \) and \( G_1 \) are replaced with their respective \( \rho \)'s, the ratio of \( \rho_1 \) and \( \rho_2 \) is specified and, consequently, the values of \( \phi_1 \) and \( \phi_2 \) are determined.

The compensation ratio of the employer’s guards to workers is found by combining Equations (11) and (13) and making use of Equations (8) and (9) with the result

\[ \frac{w(1 + \rho_2 \kappa)G_1}{w(1 + \sigma \kappa)L} = \frac{\beta_2}{\alpha \phi_1 (1 - \lambda)} \]  

When the ratio of \( G_1 \) to \( L \) is replaced with the ratio of \( \rho_1 \) to \( \sigma \), and the condition is imposed that \( \rho_1 \), \( \rho_2 \), and \( \sigma \) must sum to unity, a quadratic equation in \( \sigma \) arises, only one of whose roots is positive. Since all factor ratios are determined, the solution to the entire problem is determinate subject to a scaling factor provided by \( M \).

In three special cases, the solution to Equation (15) is easily calculated. Suppose that we designate the right-hand side of
Equation (15) as $\Gamma$. First, in the case of perfectly elastic supply of labor to the arena, the value of $\kappa$ is zero and the ratio of the firm’s guards to workers is directly proportional to $\Gamma$. Second, in the limit when $\kappa$ approaches infinity, the ratio of the firm’s guards to workers goes as the square root of $\Gamma$. Third, in the case where the elasticity of supply of workers, $\sigma_\kappa$, is unity, and $\rho_1$ and $\rho_2$ are equal, Equation (15) reduces to

$$\frac{w(1+\kappa)G_i}{wL} = 4\Gamma$$

(16)

In this case, $\kappa$ is no longer an independent variable. If we express the ratio of the firm’s guards to workers in terms of $\kappa$, Equation (16) becomes, when wages are equal,

$$\kappa = \sqrt{8\Gamma + 1}$$

(17)

EXAMPLE 1. A COAL CAMP

A small coal operator in an isolated “hollow” is likely to face a perfectly elastic output demand curve and a perfectly inelastic labor supply curve to the hollow. Let us suppose that the wages of all occupations are the same, the value of $\beta$ for both parties is 1.5, and the value of $\alpha$ is 1.00.

Let us start with the union absent. We suppose that the working population in the hollow is 1,000. The coal company employs all 1,000 as workers who produce 1,000 tons of coal, all of which is pure economic rent to the coal company, as illustrated in Figure 1. The value of rent to the coal company is $OABC$. There is no welfare loss because the coal company has captured the entire economic rent and the wage is negligible.

We now introduce the union. It begins to hire guards and the company does the same. At equilibrium, equation (15) is

$$(1-\sigma)^2 = 1.5,$$

(18)

the solution of which is

$$\sigma = \frac{-2 + 2\sqrt{6}}{10} = 0.290.$$

(19)

For a population of 1,000, there are 290 workers, 355 guards for the firm, and 355 guards for the union. Total output is 290 tons of coal, and value extracted by the union is equivalent to 290 tons of coal. The equilibrium is shown in Figure 2. The coal company earnings are represented by $OA'B'C'$. The union captures the same amount, represented by the area $C'B'AB'C'$. The welfare loss is $C'B'B'C$ and is equivalent to 420 tons of coal. The welfare loss occurs because coal production is given up when guards are hired and there is not a corresponding match in the compensation of guards.

EXAMPLE 2. PROFESSIONAL SPORTS

In a professional sports scenario, such as hockey, where a monopsony organization of club owners faces a monopoly players’ union, both sides have market power. It will be assumed that the players’ supply curve has unit elasticity. Therefore, Equation (17) can be used.

We will set the value of $\lambda$ to 0.50, $\alpha$ to 1.00, and $\beta$ to 1.50, in which case the value of $\Gamma$ is 3, and the positive root of Equation (17) is 5. Workers therefore constitute one-fifth, or the inverse of $\kappa$, of the labor force.

Let us suppose we have 5,000 individuals employed. Since labor is homogeneous, all can aspire to be hockey players, but that is not to be because there is insufficient demand. The equilibrium will have 1,000 players, 2,000 guards for the club owners, and 2,000 guards for the players’ union. The scenario appears in Figure 3.

The clubs’ gross earnings is $1,000.00, which is represented by the area $OAB'C'$. The quantity captured by the union is identical and represented by $C'B'AB'C'$. The marginal revenue product of the players is $0.25 and their wage is $0.125. Of the $125.00 wages received by the players, the area $OGF'$, or $67.50, is rent.

The total rent received by all occupations of labor is $OGFC$, and the compensation of guards is the parallelogram $OF'FC$. The relationship of the elasticity of the supply of workers and the elasticity of supply of all labor is such that the supply curves for both have the same slopes.

If the professional hockey market was purely competitive, there would be approximately 4,900 hockey players, some leaving because of the slightly lower wage, receiving a little more than $0.1117 as their wage, as shown at point $E$.

The clubs pay out $250.00 to their guards, so the net profit for the clubs is $1000.00 less $125.00 less $250.00 or a profit of $625. The players pay a like amount to their guards, and their net rent is $67.50 plus $1000.00 less $250.00 or a profit of $817.50.

The $500.00 received by the guards is pure rent to them. The social surplus is $1942.50.

The net change to social surplus because of the establishment of the union can be represented graphically as the public good created by the union (area $C'B'AB'C'$) less the loss of producer and consumer surplus for hockey players (area $C'B'EC$). The value of the public good, as has been said, is $1000.00. The approximate value of the lost consumer and producer surplus is $660 so in this scenario there is a welfare gain of about $340.
IMPLICATIONS FOR FURTHER STUDY

The bilateral monopoly model with force results in concessions for labor equal to the value of output, but at the cost of loss of output. The loss arises not because of the amount paid to guards (for their income is part of social surplus) but because of the number of guards hired. A way to reduce the loss of output is to increase the cost of hiring guards. If each side scales down its hiring of guards, each side gets the same share of winnings as before, but with fewer guards hired, more labor is released for production of the output good. The social surplus is increased, and can result in a net increase compared to the scenario in which there is no union.

Suppose we introduce a set of government constraints that include forced and enforceable negotiations of employer and union. In effect, formalized violence. Each side replaces its low-wage guards with high-wage negotiators. Input elasticity of guard violence is replaced with input elasticity of negotiating skills. If negotiators are proxies for guards, this model predicts that the proportion of winnings of employer and union should be predictable from their compensation of negotiators.
Figure 1. A Coal Camp With No Union

Figure 2. Coal Camp Equilibrium With Union
Figure 3. Professional Sports Equilibrium With Union

REFERENCES